1.0 INTRODUCTION

The objective of this experiment was to determine the fatigue strength of reinforced concrete subjected to complete reversal loading. The knowledge of these fatigue properties, which is sparsely treated in the current literature, will help in the design of reinforced concrete structure, subjected to a high intensity sound field or any form of severe vibration.

2.0 TEST SPECIMEN

Forty-five 3.25" x 1" x 25" reinforced concrete beams were cut out from three 24" x 1" x 76 light weight concrete roof slabs. Manufactured by the Alabama Cement Tile Company. The slab manufacturer provides the following specifications.

Trade Name: Alaslab
Density: 12 lb/sq. ft.
Allowable load: 60 lb/sq.ft.
Age: More than 90 days in air
Static Ultimate Tensile Strength: 730 psi, bending.
Reinforcement: 4 x 4 - 14 gage welded mesh fabric; placed at center of slab; not rusted.
Aggregate: Light weight; high limestone content; maximum size not greater than 3/8".

These beams were simply supported at both ends in all tests (see Figure 1 and 2) making the effective beam length, L, equal to 23 1/2". Metal film strain gages (Budd Company, type C6-161) were attached to the top and bottom surfaces of the beams, mid-span between the supports. These gages were first attached to small pieces of steel shim stock (3/1000" thick) which in turn were glued to the concrete surface by an epoxy cement of high bonding strength.

Approximately 20 percent of the concrete specimens obtained from the three original roof slabs were defective for one of the following:

(i) non-uniform manufacturing process.
(ii) initial crack.
(iii) cutting fault.
(iv) mishandling.

3.0 TEST METHOD

Seven beams were tested statically in order to determine the tensile strength of the specimens, the rest were tested dynamically with complete reversal loading in order to establish a stress-cycle (S-N) curve for the material.

In the static test, a beam was loaded in such a way that it failed at a section of constant bending moment. This was obtained by the use of a short piece of a 5" WF channel placed at the middle position of the concrete beam (see Figure 2). The ultimate static strength is calculated by the usual stress formula; tensile stress = Mc/I

where $M = \frac{p}{2} \left(\frac{L}{2} - a\right)$

- c = Distance from neutral axis to the outer-most "fibre", $= \frac{3}{4}$ .5"
- I = Area moment of inertia $= 3.25 \times 1^3/12 = 0.274$ in^4
- p = Load required for failure of beam
- L = Length of beam between supports
- a = Half the depth of the WF channel = 2.5"

The strain recorded during both static and dynamic tests was quite low compared to the theoretical value. The measurements were greatly affected by localized internal and/or external cracks. The use of strain gages to measure the strain levels, experienced by the specimens under test, was therefore abolished.

In the dynamic test, a beam was simply supported at both ends. The supports were then secured to the table of an electromagnetic vibrator (an M B Model C-25 HH exciter). The test set-up and the equipment used are shown in Figures 3 and 4. The inputs to the exciter, namely, the acceleration, frequency and wave form, can be controlled. The beam specimen vibrated in this fashion went through a tensile and compression stress cycle in each vibration cycle. The maximum stress experienced by the test specimen was at the outer-most fibre of the specimen, and was a function only of the applied bending moment. The bending moment will be shown to be a function of the relative acceleration of the beam with respect to its vibrating supports. Thus, any desirable stress level at the outer-most "fibre" of the beam could be controlled by the sinusoidal input to the exciter.

The data from actual test results show that the fundamental resonant frequency of most beams was 108 cps. However, to avoid difficulties that were encountered in resonant frequency testing, the beam specimens of this experiment were vibrated at a fixed frequency between 80 and 100 cps. A full report will be issued in the near future to show the validity of this stress-controlling method and the off-resonant vibration technique.

A beam thus vibrated in the dynamic test possesses the same vibration characteristics, relative to its supporting foundation, as that of a pinned-pinned beam of fixed supports, subjected to forced vibration by an oscillatory, uniformly distributing load applied along the entire length of the beam. (This statement will be proven in the full report. Notice that in both cases, the generalized forces are the same). The deflection, $y(x,t)$, of a beam with pinned-pinned supports, in its fundamental mode, is

$$y(x,t) = Y_c \sin \pi \frac{x}{L} \sin \omega t$$

where
- Y_c = maximum deflection of beam
- x = span wise coordinate
- $\omega = 2\pi f$
- f = excitation frequency, cps.

The acceleration can be expressed as

$$\ddot{y}(x,t) = \frac{\partial y \, cx,t)}{\partial t} = -\omega^2 Y_c \sin \pi \frac{x}{L}$$

$$= -\omega^2 Y(x, t)$$

or $$y(x,t) = -\ddot{y}(x,t) \frac{1}{\omega^2}$$

The maximum bending moment is

$$M_{max} = E I \left| \frac{\partial^2 y(x,t)}{\partial x^2} \right|_{max}$$

$$= -E I \left(\frac{\pi}{L}\right)^2 \; y(x,t) \; max$$

$$= E I \left(\frac{\pi}{L}\right)^2 \; \ddot{y}(x,t) \; max/\omega^2$$

$$= E I \; \ddot{y}(x,t) \; max/(2 f L)^2$$

or $$\ddot{y}(x,t) \; max = \frac{4 f^2 L^2}{E I} M \; max \tag{1}$$

where E = Young's modulus of elasticity
$= 2 \times 10^6$ psi.

It was found in the static tests that the average tensile strength of the concrete specimens was 730 psi, with an average bending moment of 402 in-lb. The ultimate strength of these specimens, in the dynamic tests was therefore assumed to be the same. However, since acceleration of the beam relative to its supports was to be controlled in the dynamic test, the ultimate strength of the specimen could be expressed in a more convenient form of acceleration, G_{ult} which would produce a bending moment of 402 in-lb at the first half cycle of vibration. From equation (1) we have:

$$G_{ult} = \frac{\ddot{y} \; (x,t)_{max}}{g} = \frac{4 f^2 L^2 \; M_{max}}{E I g}$$

$$= \frac{4 \times (f)^2 \times (23.5)^2 \times 12}{2 \times 10^6 \times 3.25} \times 402 \quad \left(\frac{1}{386}\right)$$

$$= 4.2 \times 10^{-3} \times (f)^2, \text{ g's}$$

At the test frequencies of 80, 90, 100 cps

G_{ult} = 27.1 g's (peak, at 80 cps)
G_{ult} = 34.4 g's (peak, at 90 cps)
G_{ult} = 42.4 g's (peak, at 100 cps)

4.0 TEST RESULTS

Static Tests:

Run Number	Load, P, at failure, lb
1	79
2	88
3	79
4	94
5	75
6	101
7	96
	87.4 (average)

Thus, average moment $= \frac{P}{2} \left(\frac{L}{2} - a\right)$

$= 402$ in-lb

average stress $= \frac{Mc}{I}$

$= 730$ psi , tension

Dynamic Tests:

Run No.	Freq. cps	Acc.(peak) g's	% Modulus of Rupture *	Time of run, sec.	No. of Cycles at Rupture
27	100	27.5	65	120	12,000
32	98	23	67	6	540
33	80	19	70	130	10,400
34	80	17	63	80	53,400
35	80	22	81	2	160
36	80	20	74	230	23,000
38	80	20	74	235	23,400
39	80	26	96	.5	40

* Ratio of applied stress to the ultimate stress both computed by the formula Mc/I where M is the failing bending moment.

Figure 5 shows the plot of percent modulus of rupture vs fatigue life cycle from the above results.

Figures 6 - 10 show some typical visicorder records of the accelerations measured on the beam tested.

CONCLUSION

1) The stress-cycle (S-N) plot of the test results indicates that the endurance limit of the specimen is about 60 percent.

2) All specimens failed in concrete under tension. The reinforcing steel at the middle of the beam appeared to add no tensile strength to the specimens.

3) The resonant frequency of the beam specimens decreased continuously as the load cycles increased. The decrease might be due to:

   (i) readjustment of end fixity
   (ii) the increase of internal damping
   (iii) internal and/or external cracks.

4) The dynamic magnification factor, Q, and therefore the resonant response of the beam, decreased continuously as load cycles increased. Q dropped from an initial value of 8 or 9 to 2 or 3 at the ends of some longer tests. The Q measurements were conducted by low level resonant scans at intervals during dynamic testing. The decreasing of Q indicated the increase of the internal damping of the material.

5) The continuous changing of resonant frequency and Q, and the in-phase or out-of-phase problems of response of beam to the excitation made resonant frequency

fatigue vibration testing difficult. Off-resonant frequency testing is therefore more suitable for this experiment.

6) The test specimens were not manufactured uniformly, and the material was not homogenous. Some beams, for example, failed in static test at a much lower stress than expected. Some of the tests, both static and dynamic, were therefore discarded if there was a good reason to believe that the beams were abnormal.

7) It was very difficult to determine the exact load cycles of the specimens at high percentage of modulus of rupture in the dynamic tests. The acceleration of these tests could not be brought to the desired level in a very short period of time.

8) Concrete beam specimens, tensile - reinforced at the top and bottom of the specimens, are desirable for further study of the fatigue properties of concrete.

9) Random vibration testing should be included in a further study.

INSTRUMENTATION

The instrumentation used for the tests is listed below:

1) One B and K Automatic Vibration Exciter Control, Model 1019.
2) One M B Vibrator, Model C25 HH, Type A, M. B. Mfg. Company.
3) One Electronic Counter, Hewlett Packard.
4) One Wyle 10KW Power Amplifier.
5) One Honeywell Visicorder, Model 1508.
6) One Oscilloscope, Type 545A, Tektronix, Inc.
7) One Strain Gate Indicator, Model W H 1, Strainsert Company.
8) One Endevco Accelerometer, Model 2213.
9) Three Endevco Accelerometers, Model 2226.

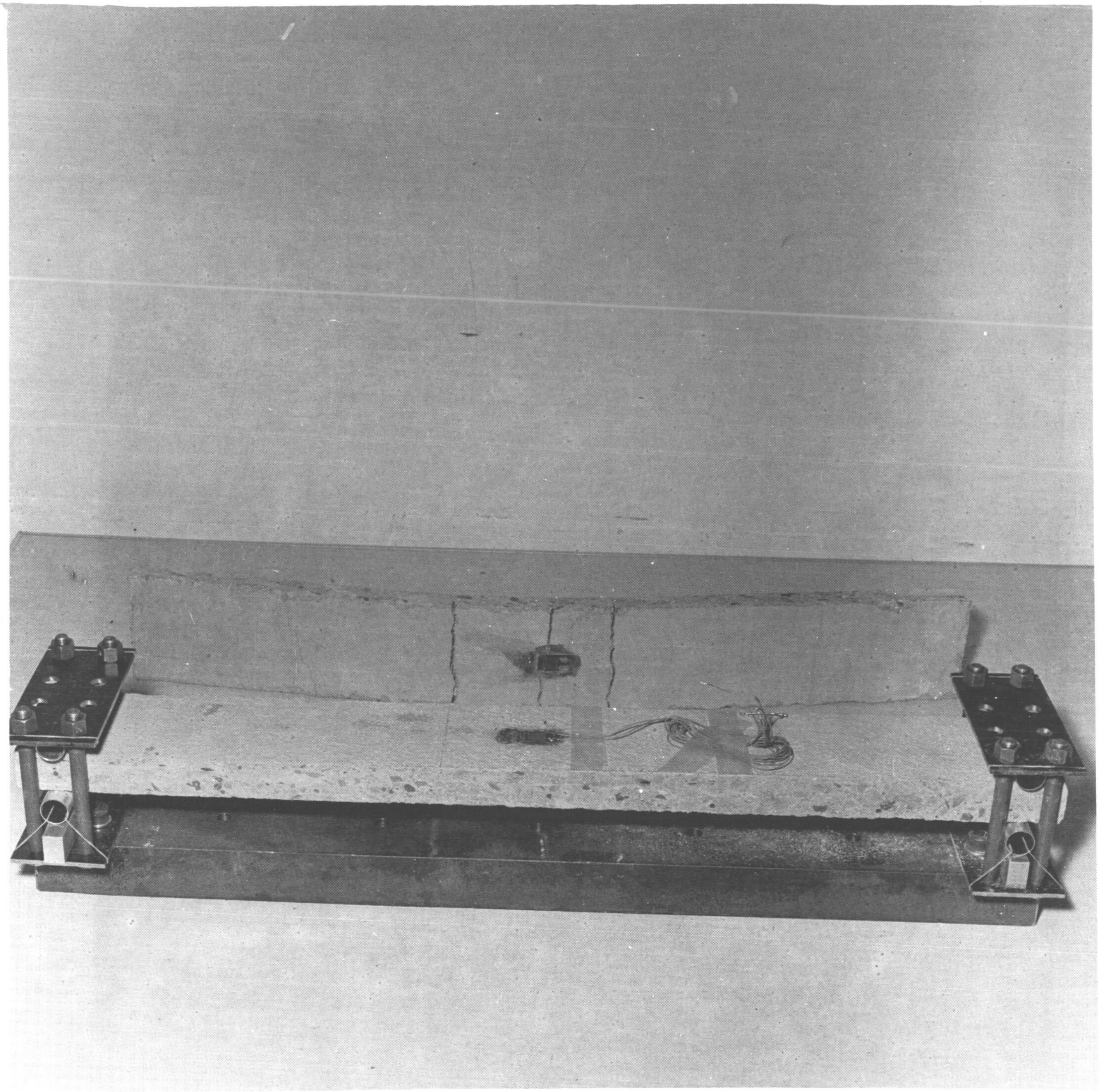

Figure 1: Concrete Beam Showing Supports in Detail and Typical Failure Mode of a Test Beam

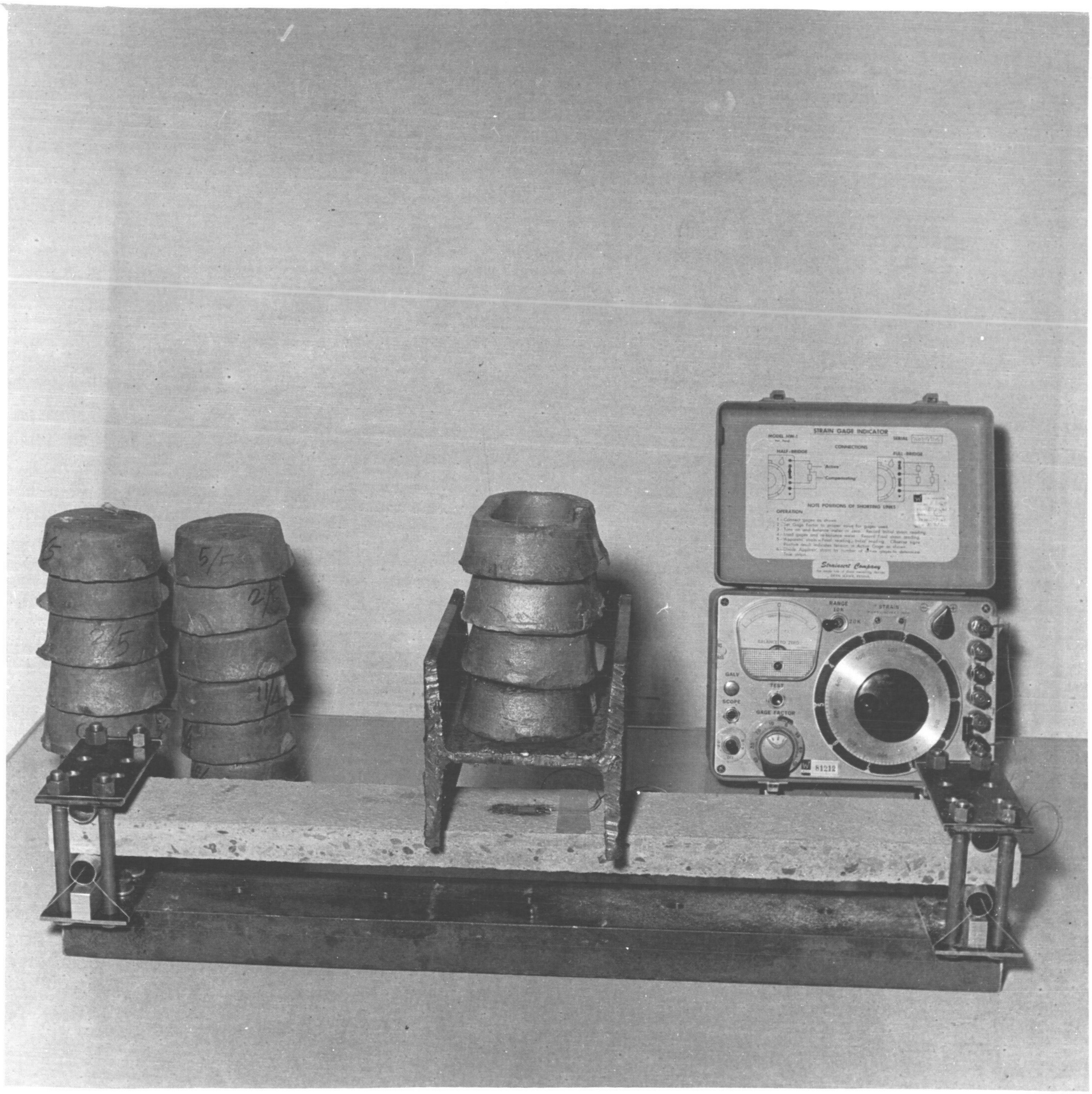

Figure 2: Simply Supported Concrete Beam Under Static Test

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Figure 3: Equipments Used in Dynamic Testing of a Concrete Beam

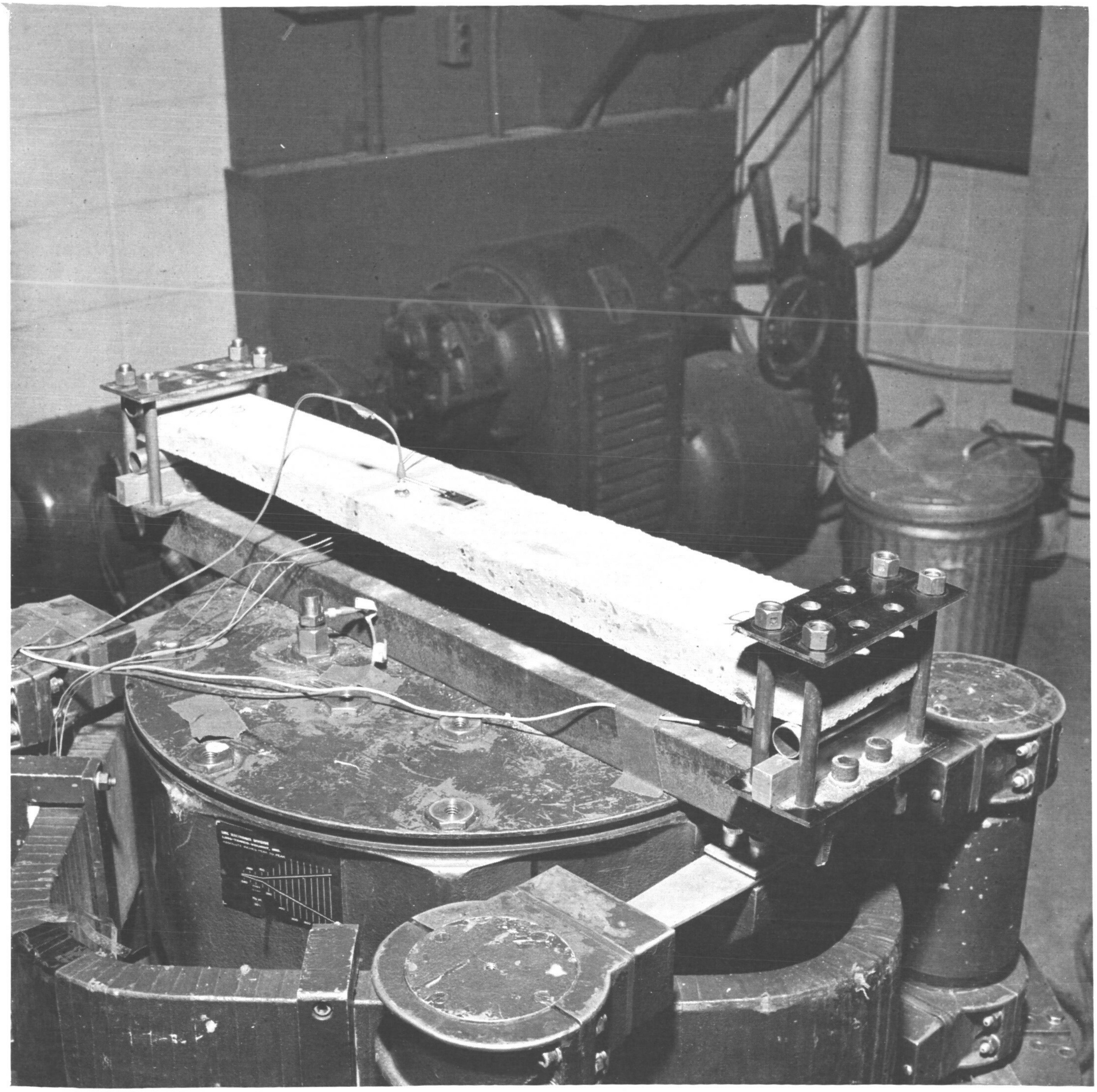

Figure 4: Dynamic Testing of a Concrete Beam Mounted on a M B C-25 HH Vibrator. Note the positions of strain gage and accelerometers.

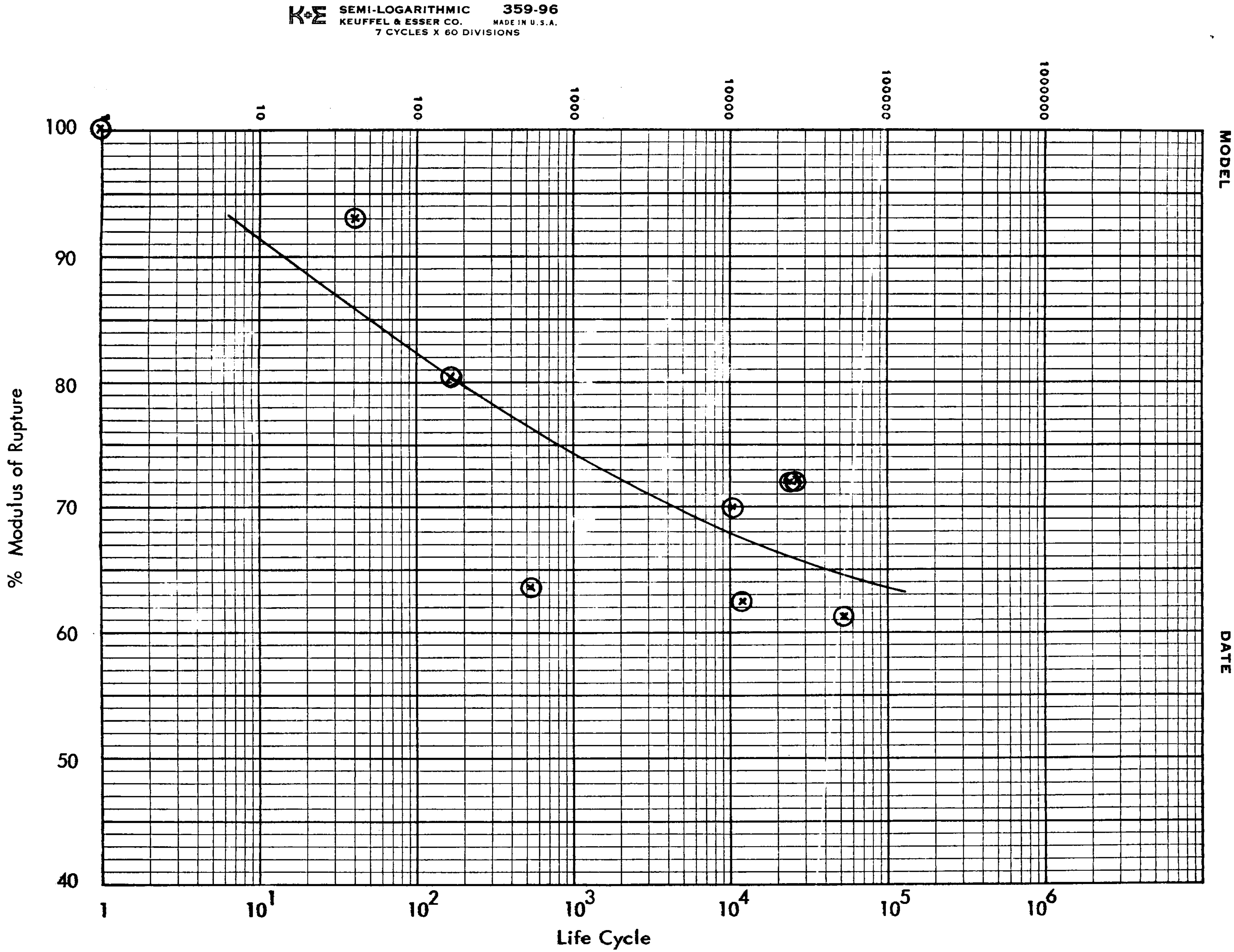

Figure 5: S-N Diagram for Light Weight Concrete Beams Reinforced at Center

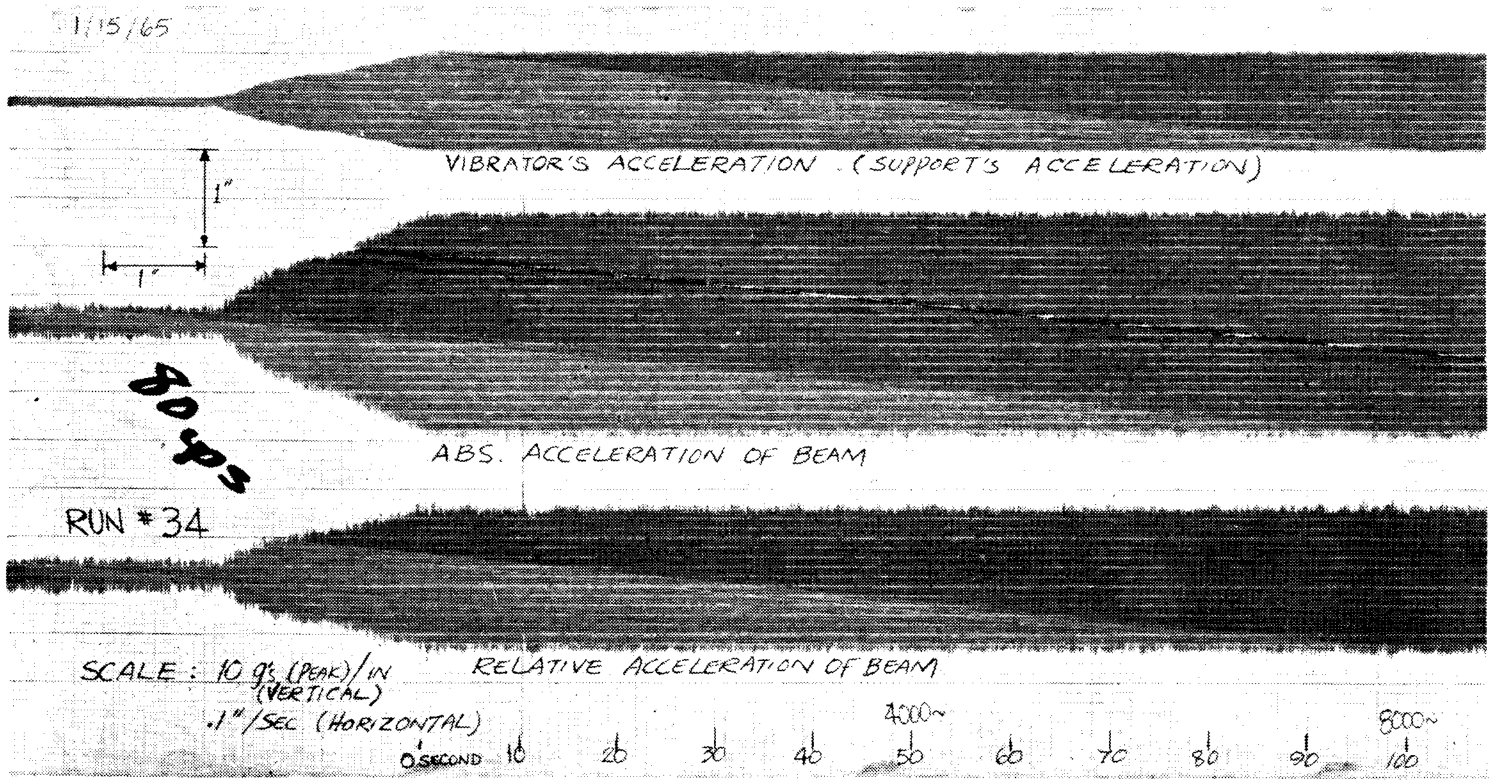

Figure 6. Run No. 34

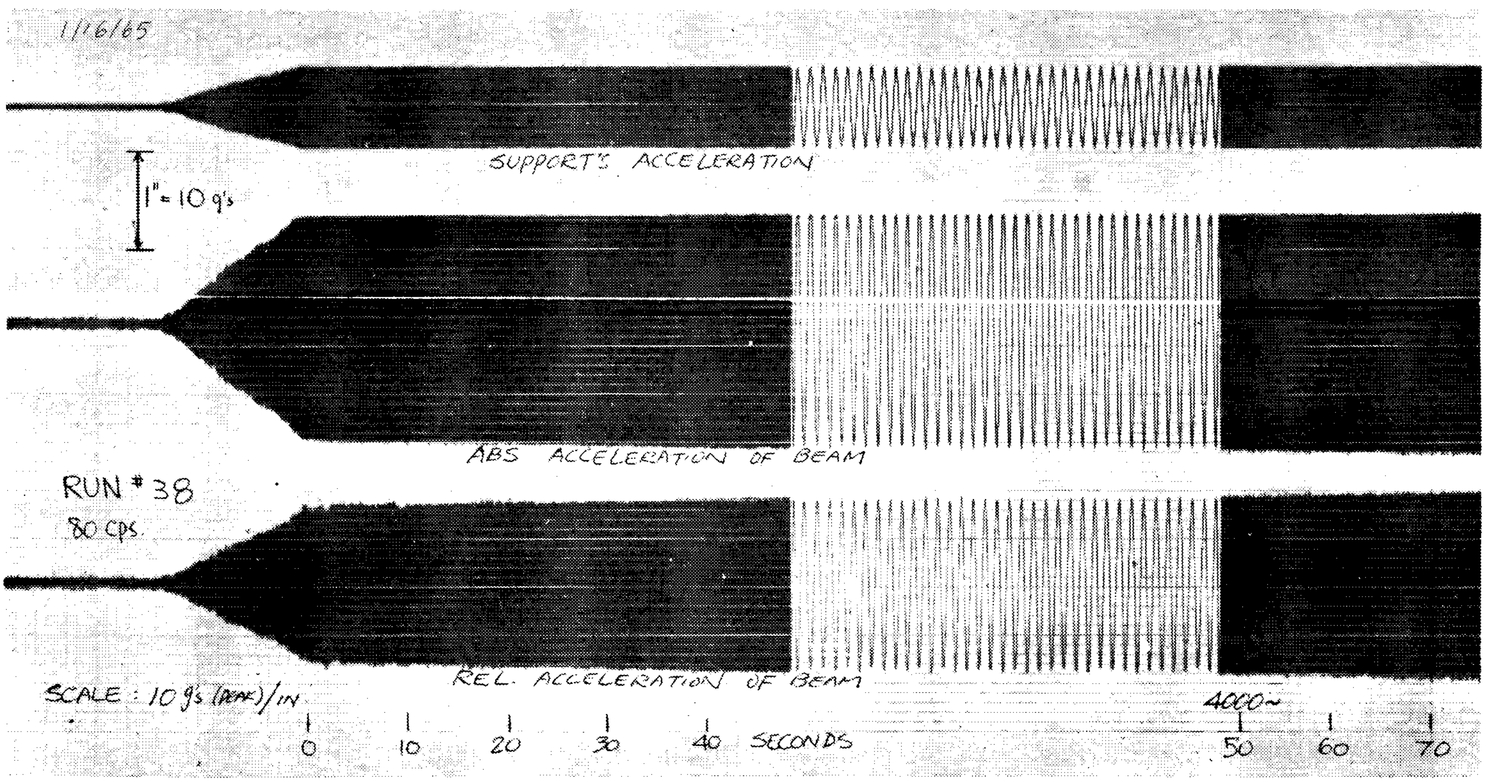

Figure 7. Run No. 38

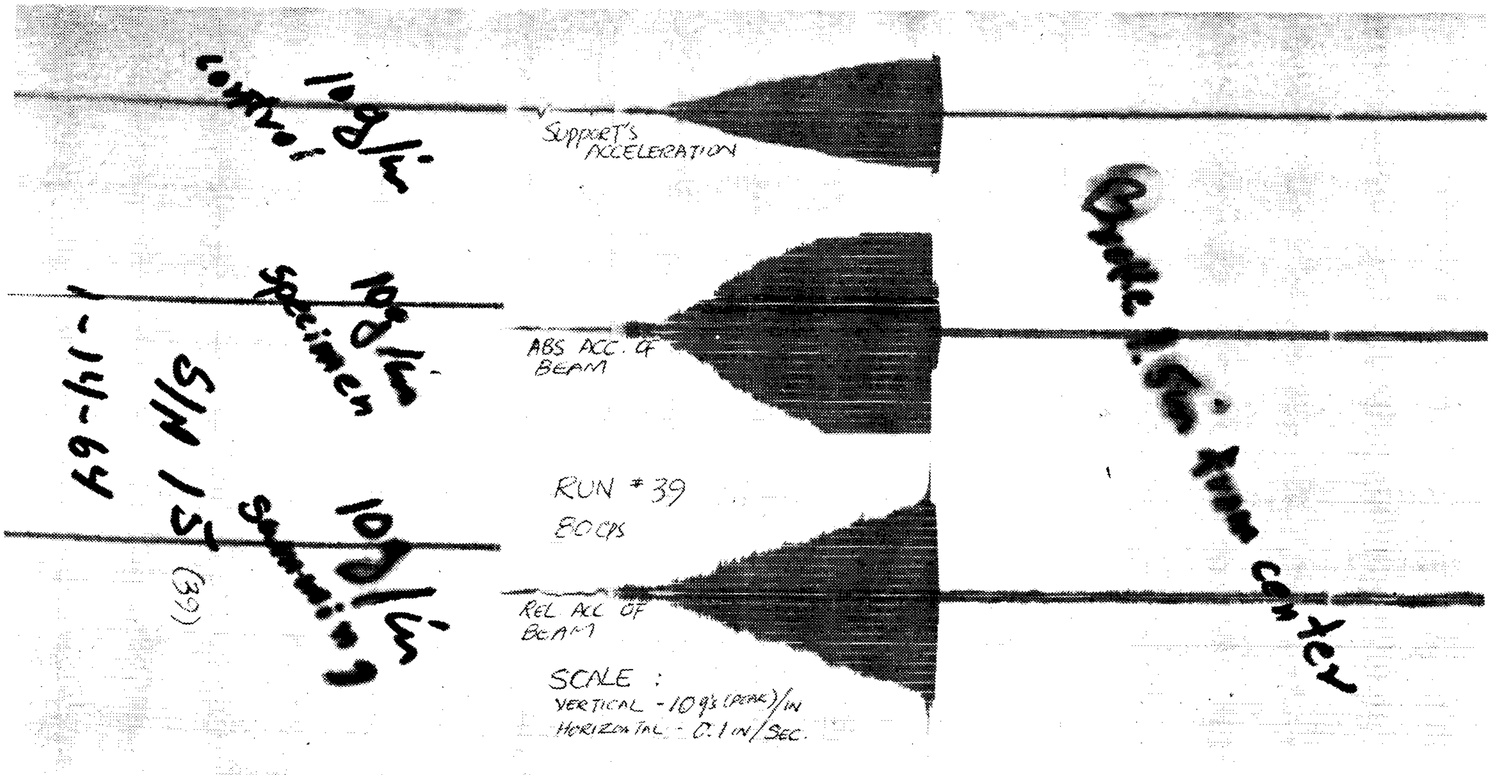

Figure 8. Run No. 39

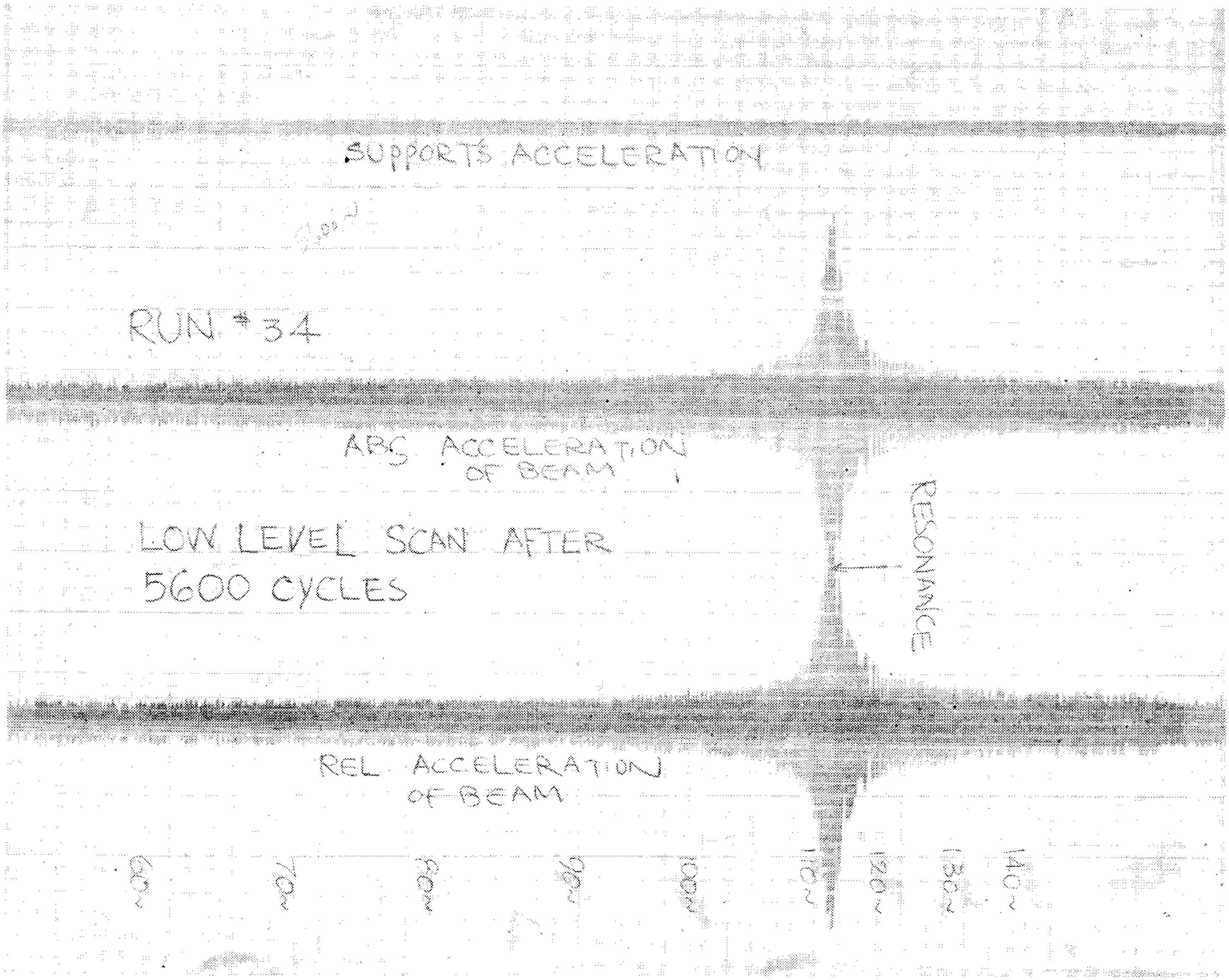

Figure 9. Low Level Scan of Run No. 34 After 5600 Cycles

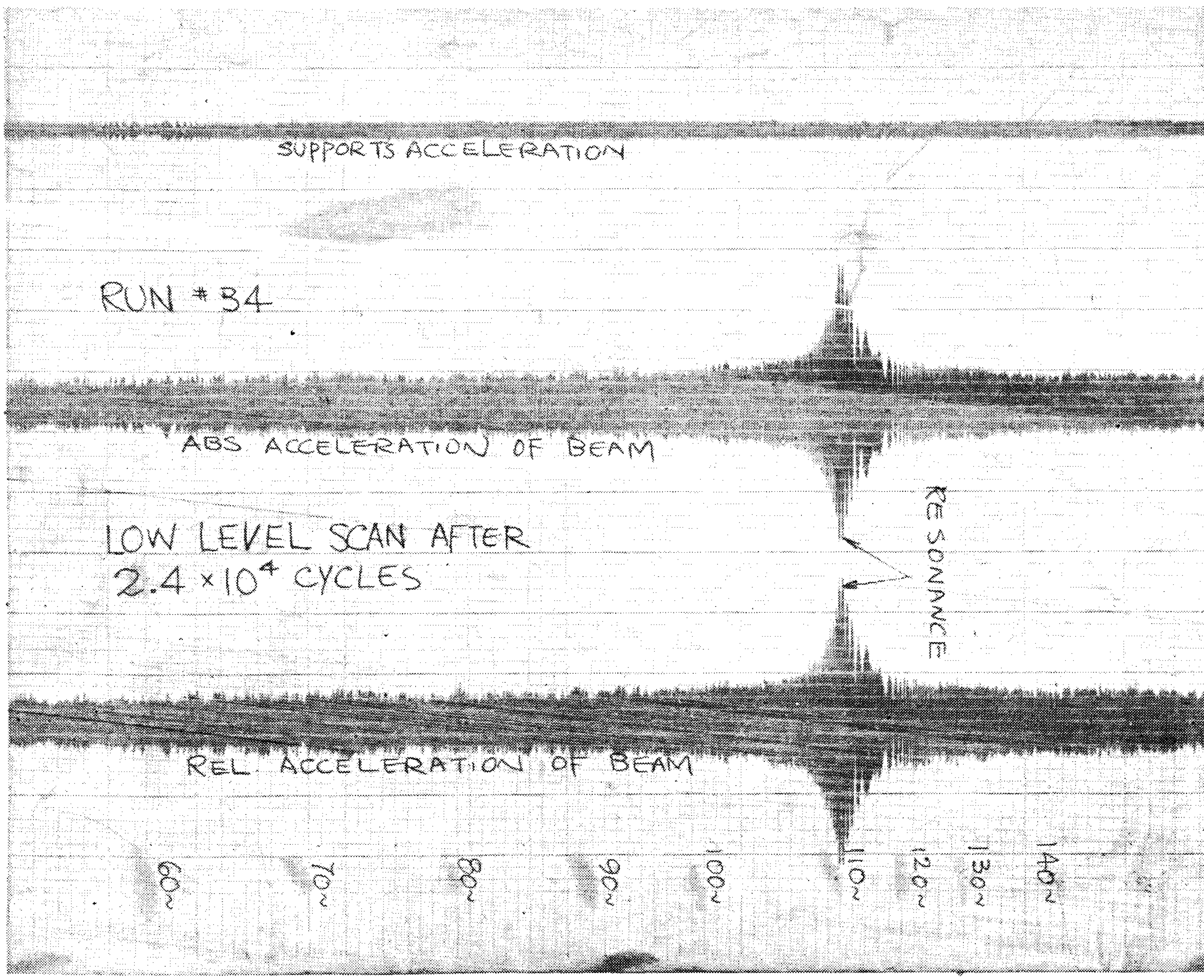

Figure 10. Low Level Scan of Run No. 34 After 24,000 Cycles